Doctorado según RD 56/2005 (CURSO 2007-08)
(www.dma.uvigo.es/master)
PROGRAMA OFICIAL DE POSGRADO (POP) INTERUNIVERSITARIO
"Métodos Matemáticos y Simulación Numérica en Ingeniería y Ciencias Aplicadas"

CURSO DE 2 CRÉDITOS ECTS

Temas Avanzados de Matemática Aplicada I

Tema: AN INTRODUCTION TO VISCOSITY SOLUTIONS: THEORY, NUMERICS AND APPLICATIONS.

Impartido por el profesor **D. MAURIZIO FALCONE**, del Departamento de Matemáticas de la *Universitá di Roma I "La Sapienza"*, ITALIA.

Contenidos

The notion of viscosity solutions, introduced by M. Crandall and P. L. Lions about twenty years ago, has provided a new tool for the analysis of nonlinear partial differential equations. This notion allows treating hyperbolic equations that have non differentiable or even discontinuous solutions as well as many (possibly) degenerate elliptic and parabolic equations. Dealing with non regular solutions is very important in many fields of applications such as control theory and front propagation.

The goal of this series of lectures is to present the basic results of this theory, analyze some approximation schemes which are particularly effective and show some applications. We will conclude giving perspectives on actual research in this field.

Part 1: An introduction to viscosity solutions.
In the first part of the course the basic notions of continuous viscosity solutions will be presented. We will start from a stationary first order model problem related to geometrical optics (the eikonal equation) and we will give existence and uniqueness results. The course will pass then to evolutive first and second order problems with a particular emphasis on the Hamilton-Jacobi equations related to the propagation of fronts.

Part 2: Approximation schemes for viscosity solutions.
The second part will deal with the numerical approximation of viscosity solutions via finite differences and semi-Lagrangian schemes. We will present some general convergence results for first order schemes and give some hints on the construction of higher order schemes. Approximation schemes for the model problems presented in the first part will be treated in detail.

Part 3: Applications.
The approximation of viscosity solutions plays a crucial role in many fields of applications. We will present, in particular, several examples related to control/game problems and front propagation. Several aspects of the implementation of the schemes (e.g. boundary conditions, acceleration methods) will be discussed.

Basic references

1. M. Bardi, I. Capuzzo Dolcetta, *Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations*, Birkhäuser, Boston, 1997.
2. G. Barles, *Solutions de Viscosité des Equations de Hamilton-Jacobi*, Springer-Verlag, Berlin (1994).
3. L. C. Evans, *Partial Differential Equations*, Graduate Studies in Mathematics, AMS, (19)1998.
4. R. Fedkiw, S. Osher, *Level Set Methods and Dynamic Implicit Surfaces*, Applied Mathematical Sciences, Springer-Verlag, New York, (153) 2003.
5. M. Falcone, *Numerical Solution of Dynamic Programming Equations*, Appendix A of the book [1], pp. 471-504.

6. M. Falcone, R. Ferretti, *Semi-Lagrangian Schemes for Hamilton-Jacobi Equations*, discrete representation formulae and Godunov methods, Journal of Computational Physics, 175, (2002), 559-575.
7. J. A. Sethian, *Level Set Methods and Fast Marching Methods*, Cambridge University Press (1999).

Fechas y horario

Del 9 al 13 de junio de 2008, en el aula 10, con el siguiente horario:

Lunes 9: de 16 a 19 horas.

Martes 10: de 10 a 12:30 y de 16 a 17:30 horas.

Miércoles 11: de 10 a 12:30 y de 16 a 17:30 horas.

Jueves 12: de 10 a 12:30 y de 16 a 17:30 horas.

Lugar

Facultad de Matemáticas, Universidad de Santiago de Compostela, Campus Sur s/n, 15782 Santiago de Compostela (A Coruña).

Información

Óscar López Pouso.
Coordinador del POP en la USC.
Teléfono: 981 563 100, ext. 13228.
Dirección electrónica: oscarlp@usc.es.

FINANCIADO POR EL MINISTERIO DE EDUCACIÓN Y CIENCIA